Mathematics
August 1, 2006

Demo Exam 1

Name: _____

Instructions. (52 points) Solve each of the following questions.

(5^{pts}) **1.** An example of a fill-in question: It is well known that _____ and _____ are jointly credited as the founders of modern calculus.

($3^{\text{pts}}_{\text{ea.}}$) **2.** *True* or *False*. No justification needed.

(a) _____ If triangles have 4 sides, then all monkeys are green. Now is the time for all good men to come to the aid of their country.

(b) _____ $1 + 1 = 3 \iff \sqrt{2}$ is a rational number. Now is the time for all good men to come to the aid of their country.

(c) _____ $(\forall x)(\exists y)(xy > 1)$ (x, y real numbers). Now is the time for all good men to come to the aid of their country.

(d) _____ $(\forall x)(\exists y)(\forall z)(z(x + y) > 0)$, ($x$, y, and z real numbers).

(11^{pts}) **3.** Select the correct answer for each of the following multiple choice. There is only one correct answer.

(a) (5 pts) In what ancient year did Columbus sail the ocean blue?

(1) 1490 (2) 1491 (3) 1492 (4) 1493

(b) (6 pts) In what year did King John sign the Magna Carta?

(1) 1213 (2) 1214 (3) 1215 (4) 1216
(5) 1217 (6) 1218 (7) 1219 (8) None of these

(5^{pts}) **4.** Which of the following best describes Augustin Cauchy?

☐ He developed the Calculus while his University was closed for the plague.

☐ Given credit for first using the functional notation $f(x)$.

☐ He created the "bell-shaped curve" and first used the method of least squares.

☐ He first formulated a precise definition of the limit and continuity of a function.

☐ Gave a rigorous definition of the definite integral—an integral that now bears his name.

☐ His notation for the derivative and the integral is used even to this day.

(5^{pts}) **5.** Define a function by $h(x) = \int_{x^2}^{5} \sqrt{t^2 + 4}\, dt$. Calculate $h'(x)$.

$h'(x) =$ ☐

(7^{pts}) **6.** Consider the region bounded by the curves $x = y^{3/2}$, $y = 4$, and the y-axis. The region is rotated about the y–axis, calculate the volume of this solid of revolution.

(7^{pts}) **7.** The base of a solid S is bounded by the x-axis, the line $y = 2x$ and the line $x = 2$. Each cross-section perpendicular to the x-axis is a *rectangle*. The base of each rectangle lies on the base of the solid and the height of the rectangle is twice that of the base. Find the volume of this solid S. (*Hint*: Read the description of the solid carefully, draw a picture of the base of the solid S, draw a typical cross-section, and compute the lengths of the sides of the rectangle as a function of x.)

Recall: Use the following facts freely throughout the exam.

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n, \quad R = 1, \qquad \sin(x) = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)\,!}\, x^{2n+1}, \quad R = \infty,$$

$$e^x = \sum_{n=0}^{\infty} \frac{1}{n\,!}\, x^n, \quad R = \infty, \qquad \cos(x) = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)\,!}\, x^{2n}, \quad R = \infty$$