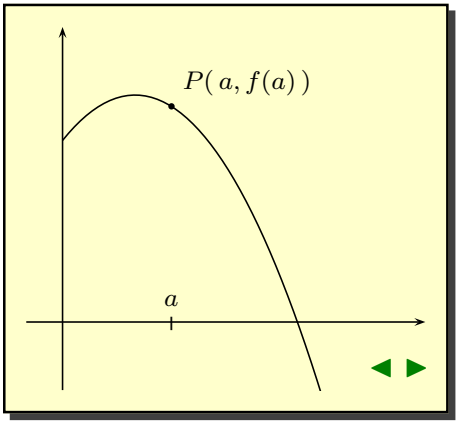
The AcroTEX Web Site, 2000

A Slide Show Demonstrating the Tangent Line Problem

D. P. Story
The Department of Mathematics
and Computer Science
The University of Akron, Akron, OH

Tangent Line Problem

Problem: Given a point $P(a, f(a))$, we want to define and calculate ...



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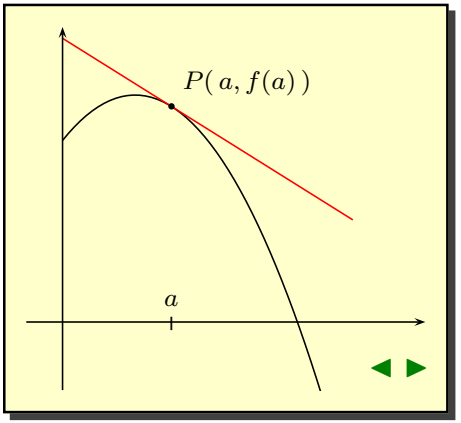

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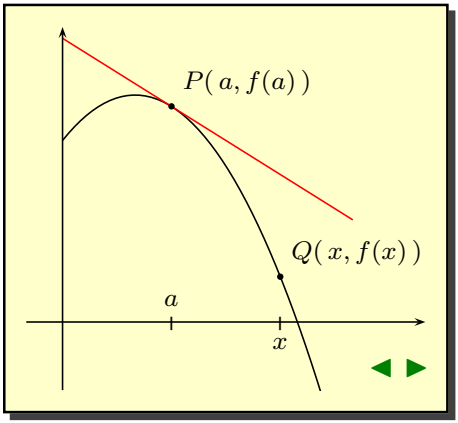

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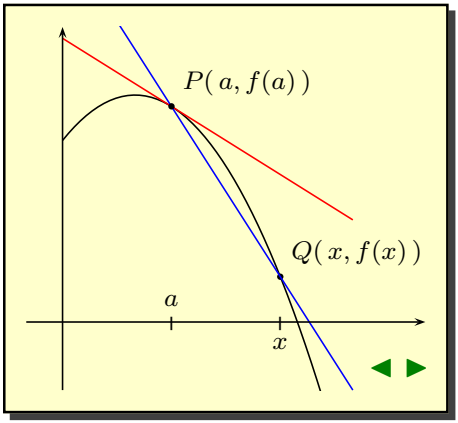

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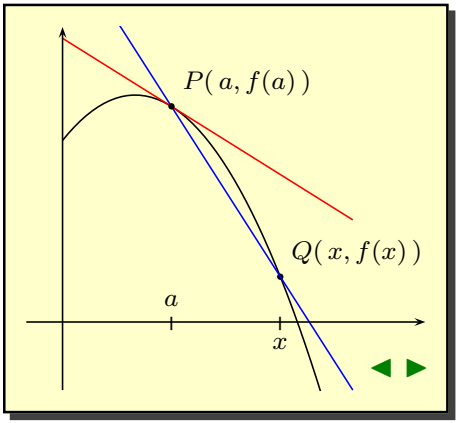

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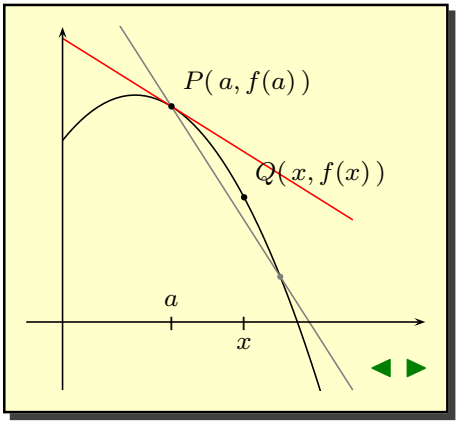

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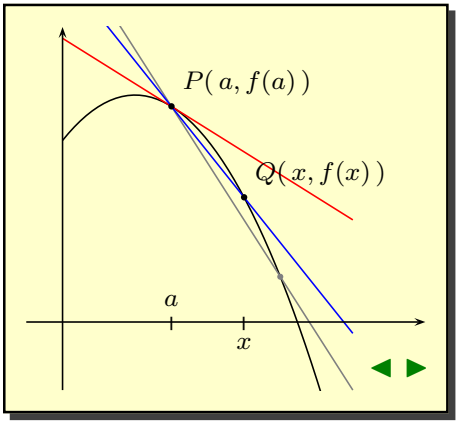

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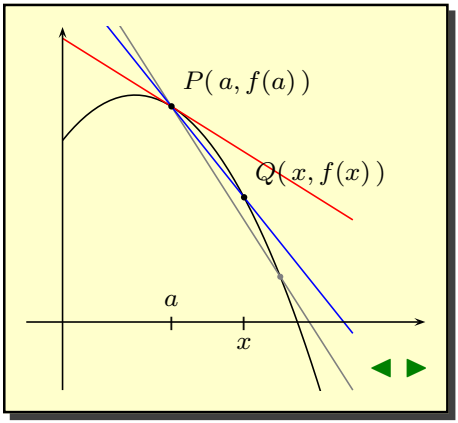

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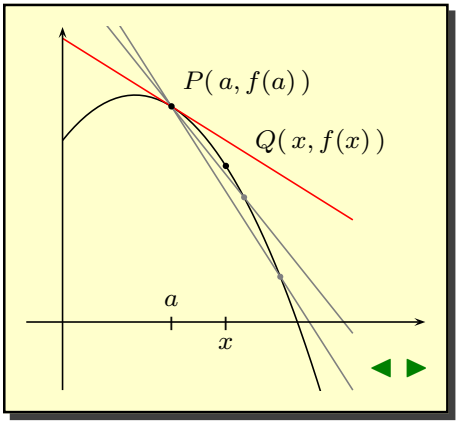

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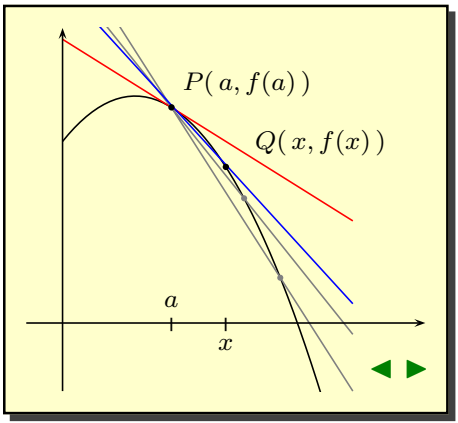

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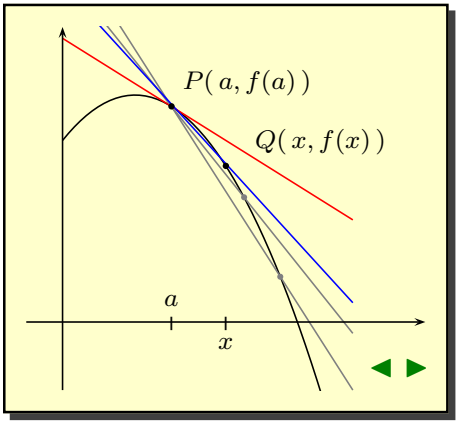

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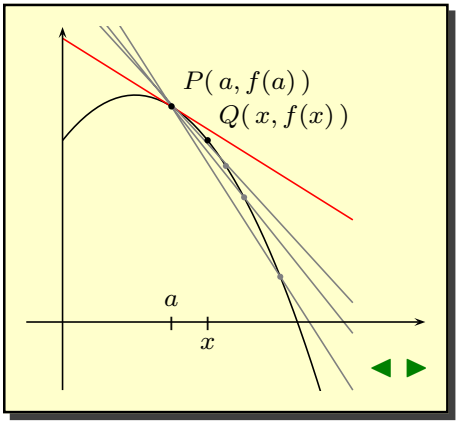

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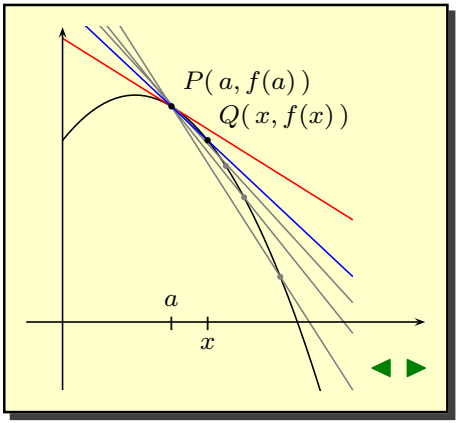

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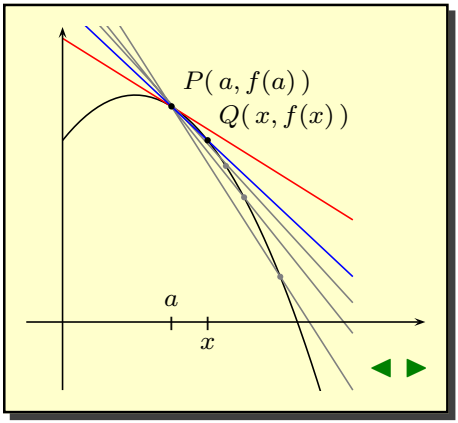

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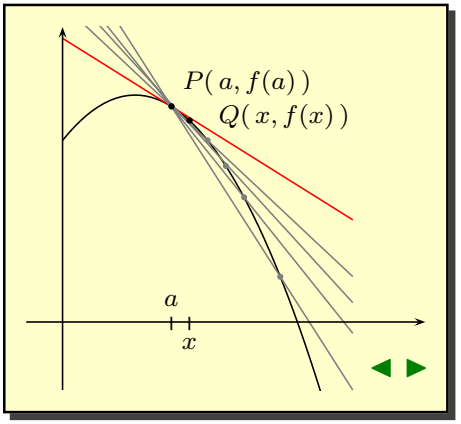

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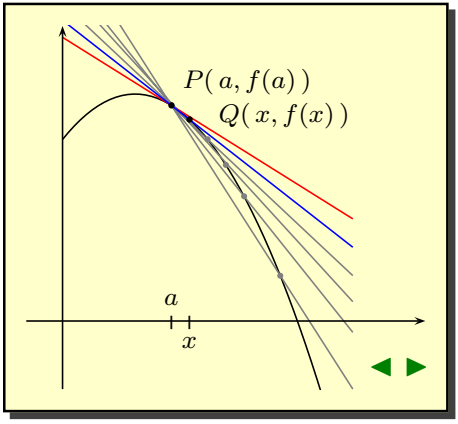

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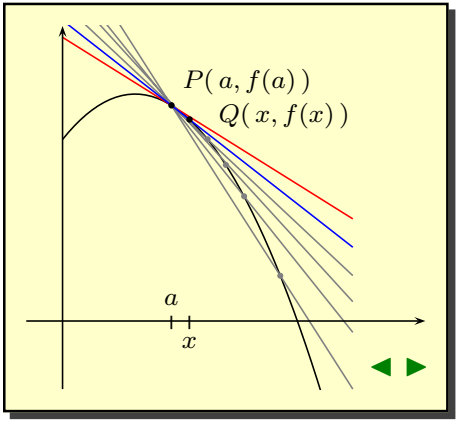

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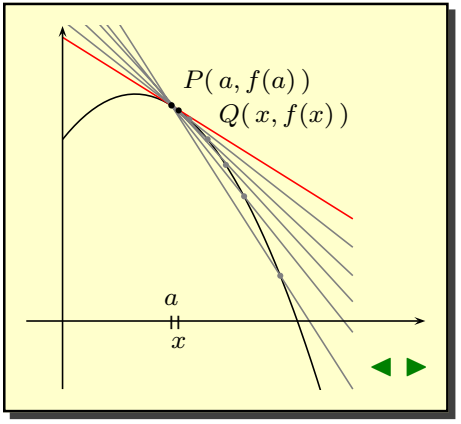

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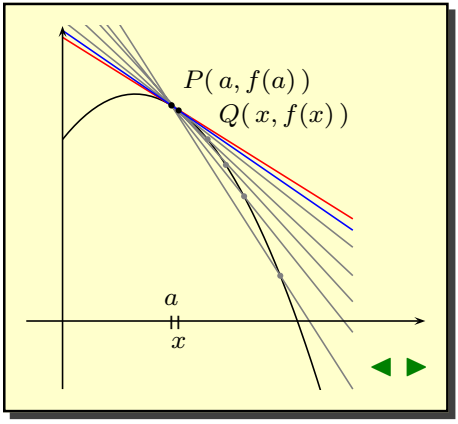

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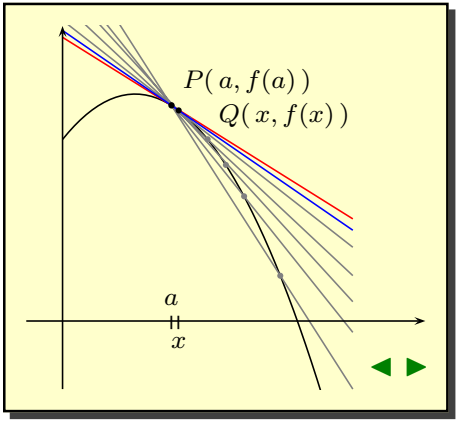

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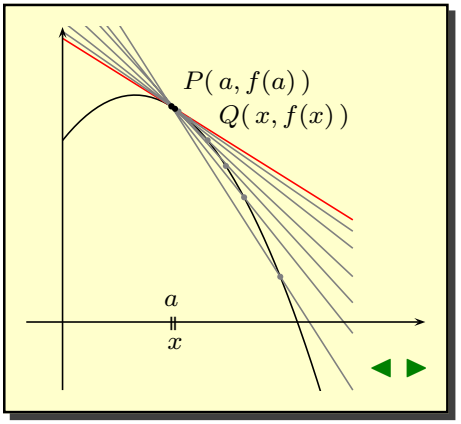

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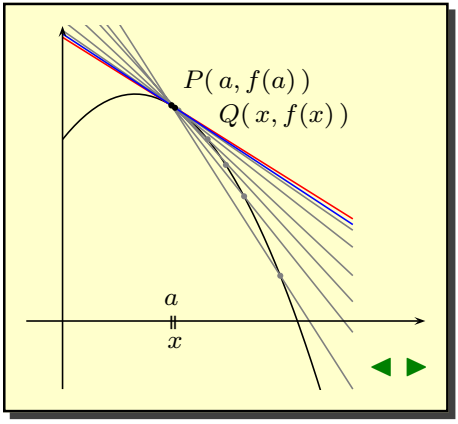

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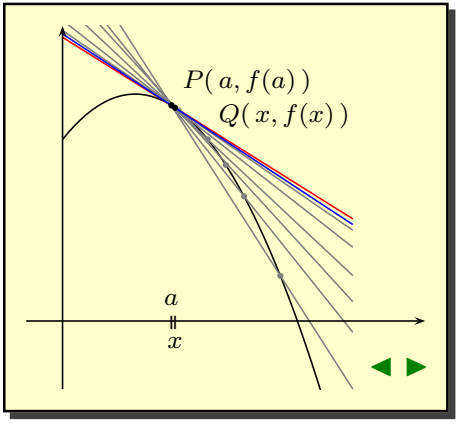

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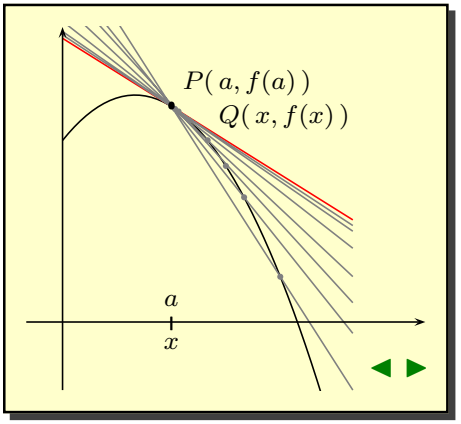

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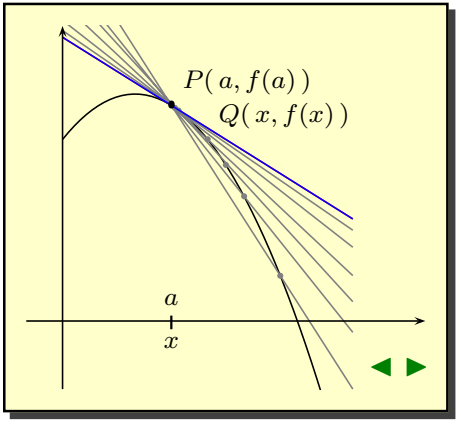

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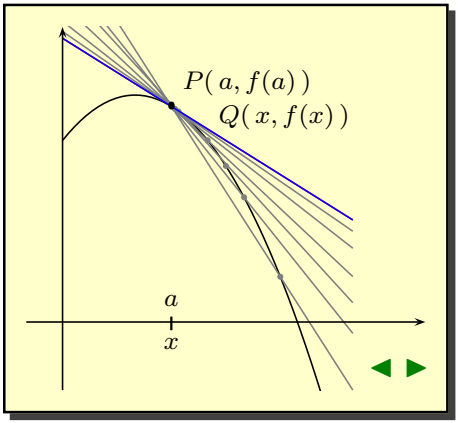

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- Repeat.
- Continue ...

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Discussion

Example: $f(x) = 5 - (x-1)^2$ and $a = 1.5$. As we choose values of x getting closer and closer to $a = 1.5$, the corresponding secant lines rotate around the point P and become more and more "tangent-like". Therefore, it is not too surprising that the slopes of these secant lines are approaching a value we would want to call "the slope of the line tangent to the graph at P".

There are more calculations for those who want to see more.

$x < 1.5$		$x > 1.5$	
x	m_{sec}	x	m_{sec}
1	−0.5	2	−1.5
1.4	−0.9	1.6	−1.1
1.45	−0.95	1.55	−1.05
1.49	−0.99	1.51	−1.01
1.499	−0.999	1.501	−1.001
1.4999	−0.9999	1.5001	−1.0001
1.49999	−0.99999	1.50001	−1.00001

The values of m_{sec} appear to be getting close and closer to -1. In this case, we write:

$$\lim_{x \to 1.5} m_{\text{sec}} = -1$$